

ÉCOLE NORMALE SUPÉRIEURE DE PARIS (ENS–PSL)

MASTER IMALiS M1

YEAR 2023–24

UE Mathematics II : What a biologist might like to know

PROGRAM

In charge	: Amaury Lambert Institute of Biology of ENS (IBENS) & Center for Interdisciplinary Research in Biology (CIRB) – Collège de France
Contact	: 01 44 32 20 32 or amaury.lambert@ens.psl.eu & 01 44 27 13 91 or amaury.lambert@college-de-france.fr
Lecturers	: AL and Mathilde André (mathilde.andre@college-de-france.fr)
Timetable	: Each session lasts 3 hours and starts either at 9 am or at 2 pm
Location	: All lectures take place in Room 324, including computer-based tutorials (CBT)
Prerequisites	: Lectures of L3 “Mathematics I : What a biologist should like to know”.

1. **Mon 4 Sep morning.** Preliminaries. Topology. Measure. Inner product.
2. **Mon 4 Sep afternoon.** Harmonic analysis (I). Fourier series. Fourier transform. Characteristic function.
3. **Wed 6 Sep morning.** Dynamical systems (I). Invariant sets, irreducible sets, attractors. Ergodic measure. Ergodic Theorem.
4. **Wed 6 Sep afternoon (CBT1).** Harmonic analysis (II). Introduction to programming in Python. Fourier calculus. Central Limit Theorem.
5. **Mon 11 Sep morning.** Dynamical systems (II). Lyapunov exponent. Canonical examples : Bernoulli shift, logistic map.
Probability (I). Time-discrete Markov chains. Reminders, stationary distribution, hitting probability. Canonical examples : random walk, Bienaymé–Galton–Watson process, Wright–Fisher model.
6. **Mon 11 Sep afternoon.** Probability (II). Time-continuous Markov chains. Definition, Kolmogorov Equations. Transition rate, notion of generator. Canonical examples : time-continuous random walk, linear birth-death process, Moran model. Stationary probability, hitting probability.

7. **Wed 13 Sep morning (CBT2).** Dynamical systems (III). Chaotic population dynamics, May's logistic model, Lorenz attractor.
8. **Wed 13 Sep afternoon.** Probability (III). Brownian motion and stochastic differential equations. Canonical examples : Feller diffusion, Fisher–Wright diffusion.
9. **Mon 18 Sep morning.** Partial differential equations (I). Conservation law, transport equations, McKendrick–von Foerster Equation.
10. **Mon 18 Sep afternoon (CBT3).** Probability (IV). Diffusion processes in neuroscience and in ecology.
11. **Wed 20 Sep morning.** Partial differential equations (II). Heat Equation, reaction-diffusion equations, Fisher-KPP Equation.
12. **Wed 20 Sep afternoon (CBT4).** Partial differential equations (III). Simulation of a few PDEs.
13. **Mon 25 Sep morning.** Working session on project (in presence of M. André).
14. **Mon 25 Sep afternoon.** Working session on project (in presence of M. André).
15. **Wed 27 Sep morning.** Working session on project (in presence of M. André).
16. **Wed 27 Sep afternoon.** Oral presentations.